Analysis of a birth and death process with alternating rates and of a telegraph process with underlying random walk
ABSTRACT

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My thesis for the Doctoral Programme in Mathematics (November 1, 2008 - October 31, 2011) at University of Salerno, Italy, has been oriented to the analysis of two stochastic models, with particular emphasis on the determination of their probability laws and related properties. The discussion of the doctoral dissertation will be given in 20 March 2012.

The first part of the thesis is devoted to the analysis of a birth and death process with alternating rates. We recall that an extensive survey on birth-death processes (BDP) has been provided by Parthasarathy and Lenin [3]. In this work the authors adopt standard methods of analysis (such as power series technique and Laplace transforms) to find explicit expressions for the transient and stationary distributions of BDPs and provide applications of such results to specific fields (communication systems, chemical and biological models). In particular, in Section 9 they use BDPs to describe the time changes in the concentrations of the components of a chemical reaction and discuss the role of BDPs in the study of diatomic molecular chains. Moreover, the paper by StockMayer et al. [4] gives an example of application of stochastic processes in the study of chain molecular diffusion. In this work a molecule is modeled as a freely-joined chain of two regularly alternating kinds of atoms. All bonds have the same length but the two kinds of atoms have alternating jump rates, i.e. the forward and backward jump rates for even labeled beads are α and β, respectively, and these rates are reversed for odd labeled beads. The authors obtain the exact timedependent average length of bond vectors. Inspired by this works, Conolly [1] studied an infinitely long chain of atoms joined by links of equal length. The links are assumed to be subject to random shocks, that force the atoms to move and the molecule to diffuse. The shock mechanism is different according to whether the atom occupies an odd or an even position on the chain. The originating stochastic model is a randomized random walk on the integers with an unusual exponential pattern for the inter-step time intervals. The authors analyze some features of this process and investigate also its queue counterpart, where the walk is confined to the non negative integers.

Stimulated by the above researches, a birth and death process $N(t)$ on the integers with a transition rate λ from even states and a possibly different rate μ from odd states has been studied in the first part of the thesis. A detailed description of the model is performed, and the Chapman-Kolmogorov equations are introduced. Then, the probability generating functions of even and odd states are then obtained. These allow to evaluate the transition probabilities of the process for arbitrary integer initial state. Certain symmetry properties of the transition probabilities are also pinpointed. Then, the birth and death process obtained by superimposing a reflecting boundary in the zero-state is analyzed. In particular, by making use of a Laplace transform approach, the probability of a transition from state 0 or state 1 to the zero-state is obtained. Formulas for mean and variance of both processes are finally provided.

The second part of the thesis is devoted to the analysis of a generalized telegraph process with an underlying random walk. The classical telegraph process describes a random motion on the real line characterized by two finite velocities with opposite directions, where the velocity changes are governed by a time-homogeneous Poisson process (see Orsingher [2]). The novelty in the proposed model consists in the use of new rules for velocity changes, which are now governed by a sequence of Bernoulli trials. This implies that the random times separating consecutive changes of direction of the moving particle have a general distribution and form a non-regular alternating renewal process. Starting from the origin, the running particle performs an alternating motion with velocities c and $-v$ $(c, v > 0)$. The direction of the motion (forward and backward) is determined by the velocity sign. The particle changes the direction according to the outcome of a Bernoulli trial. Hence, this defines a (possibly asymmetric) random walk governing the choice of the velocity at any epoch. By adopting techniques based on renewal theory, the general form of probability law is determined as well as the mean of the process. Furthermore, two instances are investigated in detail, in which the random intertimes between consecutive velocity changes are exponentially distributed with (i) constant rates and with (ii) linearly increasing rates. In the first case, explicit expressions of the transition density and of the conditional mean of the process are expressed as series of Gauss hypergeometric functions. The second case leads to a damped random motion, for which we obtain the transition density in closed form. It is interesting to note that the latter case yields a logistic stationary density.

References

[1] Conolly B.W. (1971) *On randomized random walks.* SIAM Review, 13, 81-99.

[2] Orsingher, E. (1990) *Probability law, flow functions, maximum distribution of wave-governed random motions and their connections with Kirchoff's laws.* Stoch. Process. Appl., 34, 49-66.

[3] Parthasarathy P.R. and Lenin R.B. (2004) *Birth and death process (BDP) models with applications–queueing, communication systems, chemical models, biological models: the state-ofthe- art with a time-*

dependent perspective. American Series in Mathematical and Management Sciences, vol. 51, American Sciences Press, Columbus (2004)

[4] Stockmayer W.H., Gobush W. and Norvich R. (1971) *Local-jump models for chain dynamics.* Pure Appl. Chem., 26, 555-561.

NOTE

The thesis consists of four chapters:

Chapter 1. Some definitions and properties of stochastic processes.

Chapter 2. Analysis of birth-death processes on the set of integers, characterized by alternating rates.

Chapter 3. Results on the standard telegraph process.

Chapter 4. Study of the telegraph process with an underlying random walk governing the velocity changes.