Università degli Studi di Salerno

Dipartimento di Ingegneria Elettronica ed Ingegneria Informatica

Dottorato di Ricerca in Ingegneria dell'Informazione
X Ciclo – Nuova Serie

TESI DI DOTTORATO

The Unlucky Broker

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Anno Accademico 2010 – 2011

Abstract

This dissertation collects results of the work on the interpretation, characterization and quantification of a novel topic in the field of detection theory *-the Unlucky Broker problem-*, and its asymptotic extension.

An example revealing the origin of the name *unlucky broker* can be found in everyday life. Bernard is a broker and his goal is to suggest some good investments to his customers. He has available two data sets: one in the public domain and the other one made of certain confidential information he has received. To recommend the appropriate investments, Bernard must decide between a positive or a negative market trend. He makes his decision by setting the probability of wrongly predict a positive trend, while maximizing the corresponding probability of correctly making such prediction. Suppose that, later, Bernard is asked to refine his own decision: he must assure the probability of wrongly predicting a positive trend to be much lower than that initially chosen. Unfortunately, the unlucky Bernard has lost the files containing the confidential information, but he reminds the decision previously made. What should Bernard do? Should he simply retain the original decision, or should he ignore that and use only the available data set for a completely new decision? Or, what else?

The same problem can be also applied to the context of Wireless Sensor Networks (WSNs). Suppose that a WSN is engaged in a binary detection task. Each node of the system collects measurements about the state of the nature ($\mathcal{H}_0$ or $\mathcal{H}_1$) to be discovered. A common fusion center receives the observations from the sensors and implements an optimal test (for example in the Bayesian sense), exploiting its knowledge of the a-priori probabilities of the hypotheses. Later, the priors used in the test are revealed to be inaccurate and a refined pair is made available. Unfortunately, at that time, only a subset of the original data is still available, along with the original decision. In this example, the unlucky broker problem is that of refining the original decision, by using the new pair of priors and the surviving data.

In the first part of this thesis, we abstract the problem in statistical terms and we find the optimal detector solving the unlucky broker problem, both in the Neyman-Pearson (NP) and the Bayesian frameworks. We show that when an effective refinement is required (variation of the required false alarm probability in the NP case, or updating of a-priori in the Bayesian scenario), some decisions can be safely retained, but others require a deeper analysis. The main finding is that, given the original decision, the solution to the Unlucky Broker problem does not amount to a single threshold test.

The second part of this dissertation concerns the extension of the Unlucky Broker problem to further detection stages in presence of multiple successive refinements. In particular, we study the asymptotic behavior of the system. This problem -the Very Unlucky Broker- can be assimilated to an emergency scenario.

Consider a group of rescuers hooking into an emergency area where some disaster has just happened. They walk in single line and as they penetrate the damaged area, a gradual reduction of available information occurs, since the

communication with the outside world is partially impaired, visibility is poor, people are confused, the psychological situation of the rescuer worsen, and so forth. Accordingly, the amount of the information available to any rescuer depends on his position on the line. Any rescuer makes a decision about how to operate, by exploiting his data set and the decision of the individual behind him. Moreover, the reliability of the decision depends on the position of the rescuer in the line: the more one is closer to the core of the disaster, the more reliable the decision should be. What about the behavior of a long chain of such rescuers?

In the thesis, we formulate the problem in statistical terms and we consider a system made of n sensors engaged in a binary detection task. A successive reduction of data set's cardinality occurs and multiple refinements are required. The sensors are devices programmed to take the decision from the previous node in the chain and the available data, implement some simple test to decide between the hypotheses, and forward the resulting decision to the next node.

The first part of the thesis shows that the optimal test is very difficult to be implemented even with only two nodes (the unlucky broker problem), because of the strong correlation between the available data and the decision coming from the previous node. Then, to make the designed detector implementable in practice and to ensure analytical tractability, we consider suboptimal local tests.

We choose a simple local decision strategy, following the rationale ruling the optimal detector solving the unlucky broker problem: A decision in favor of $\mathcal{H}_0$ is always retained by the current node, while when the decision of the previous node is in favor of $\mathcal{H}_1$, a local log-likelihood based test is implemented.

The main result is that, asymptotically, if we set the false alarm probability of the first node (the one observing the full data set) the false alarm probability decreases along the chain and it is non zero at the last stage. Moreover, very surprisingly, the miss detection probability decays exponentially fast with the root square of the number of nodes and we provide its closed-form exponent, by exploiting tools from random processes and information theory.